A History of Muslim Philosophy


Mathematics and Astronomy


Chapter LXIII

MATHEMATICS AND ASTRONOMY

A INTRODUCTION

It is generally recognized that human knowledge took its organized and systematic form with the Greeks. It is equally well known that the Greeks inherited a considerable body of knowledge from their Eastern predecessors, especially the Egyptians, Babylonians, Chaldeans, and Indians. The histories of mathematics and science, written by some Western writers, however, show a gap between the period of the Greeks and the Renaissance. They give the impression that the history of science was blank for nearly one thousand years, and scientific knowledge made a sudden leap, taking a momentum in its stride. These histories ignore the fact that the intervening ages from the first/seventh to the eighth/fourteenth century constituted the era of the Arab and other Muslim peoples. The latest researches of Muslim and non-Muslim scholars are bringing to light the work of the Muslims in the various branches of knowledge throughout the Middle Ages. These researches are, however, scattered in various journals and books which are not easily accessible to the average educated person. Two good works of reference published are the *Encyclopedia of Islam* and George Sarton's *Introduction to the History of Science*. On a thorough study of the information available on the subject, one is struck by the magnitude as well as importance of the contributions made by the Muslims to the various branches of science, especially mathematics and astronomy. The magnitude of these achievements is so vast that it is giving rise to another tendency among the historians of science. It is incomprehensible to them that the Arabs who were so backward and ignorant in the centuries preceding the advent of Islam could have become so enlightened and scholarly in such a short time after adopting the new faith. One of the great exponents of this line of thought is Moritz Cantor who has written an encyclopedic history of mathematics in the German language. The chapter on the Arabs in Cantor's book begins as follows:

"That a people who for centuries together were closed to all the cultural influences from their neighbours, who themselves did not influence others
during all this time, who then all of a sudden imposed their faith, their laws, and their language on other nations to an extent which has no parallel in history—all this is such an extraordinary phenomenon that it is worthwhile to investigate its causes. At the same time we can be sure that this sudden outburst of intellectual maturity could not have originated of itself."

Labouring under this fixed idea, Cantor proceeds to attribute almost everything done by the Muslim scholars to the Greeks and other nations. We must confess that this kind of argument introduces an extremely dangerous principle in historical research, and can be employed only by one who is predisposed to demolish an exalted and established reputation. If Cantor had really investigated the cause of the "sudden outburst of intellectual maturity" of the Arabs, he would have realized that it was primarily due to the revolution caused by Islam in the whole outlook of the people. We have elsewhere described the attitude of Islam towards knowledge.1 By making it incumbent upon the believer to acquire knowledge and by enjoining upon him to observe and to think for himself, Islam created an unbounded enthusiasm for acquiring knowledge amongst its followers. The result of this revolution can be best described in the words of Florian Cajori, who says in his History of Mathematical Notation: "The Arabs present an extraordinary spectacle in the history of civilization. Unknown, ignorant, and dismembered tribes of the Arabian Peninsula, untrained in government and war, are, in the course of ten years, fused by the furnace-blaze of religious enthusiasm into a powerful nation, which in one century extends its dominion from India across northern Africa to Spain. A hundred years after this grand march of conquest, we see them assume the leadership of intellectual pursuits; the Muslims become the great scholars of their time."

It is under this stimulus of the Islamic injunction for acquiring more and more knowledge that the Arabs and other Muslim peoples turned to the learning of the various branches of knowledge, preserving and improving upon the heritage left by preceding civilizations and enriching every subject to which they turned their attention. In the following pages we give an account of their contribution in the domain of mathematics and astronomy. It may be pointed out that this is only a brief chapter in the general history of Muslim philosophy. The account will, therefore, be of a descriptive nature, shorn of all technicalities and confined to some of the fundamental ideas put forward by the Muslim peoples in the fields of arithmetic, algebra, geometry, trigonometry, and astronomy. It is neither possible nor desirable to give here an exhaustive account of the work done by each and every Muslim scholar. We have restricted ourselves to important contributions of the prominent Muslim mathematicians and astronomers.

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1 Vol. I, Chap. VIII.

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B ARITHMETIC

The Arabs started work on arithmetic in the second/third century. Their first task in this field was to systematize the use of the Hindu numerals which are now permanently associated with their names. Obviously, this was an immense advance on the method of depicting numbers by the letters of the alphabet which was universal up to that time and which prevailed in Europe even during the Middle Ages. The rapid development in mathematics in the subsequent ages could not have taken place without the use of numerals, particularly zero without which all but the simplest calculations become too cumbersome and unmanageable. The zero was mentioned for the first time in the arithmetical work of al-Khwarizmi written early in the third/ninth century. The Arabs did not confine their arithmetic to integers only, but also contributed a great deal to the rational numbers consisting of fractions. This was the first extension of the domain of numbers, which, in its logical development, led to the real, complex, and hyper-complex numbers constituting a great part of modern analysis and algebra. They also developed the principle of error which is employed in solving algebraic problems arithmetically. Al-Biruni (393-432/973-1044), Ibn Sina (370-428/960-1037), Ibn al-Samh (d. 427/1035), Muhammad ibn Tussin al-Qari (d. 410/1019 or 420/1029), Abu Sahl al-Sijzi (c. 340-415/c. 953-c. 1024) are some of the arithmeticians who worked on the higher theory of numbers and developed the various types of numbers, such as:

(i) Tamas (perfect numbers), i.e., those which are equal to the sum of their divisors, e.g., 6 = 1 + 2 + 3.
(ii) Metu'sabāls (equivalents), i.e., two numbers, the sum of the divisors of which is the same, e.g., 39 and 55: 1 + 3 + 13 = 1 + 5 + 11.
(iii) Mosālikhās (amicable numbers), i.e., two such numbers in which the sum of the divisors of one equal the other, e.g., 220 and 284:
   \[1 + 2 + 4 + 71 + 142 = 220\]
   \[1 + 2 + 4 + 5 + 10 + 11 + 22 + 24 + 44 + 55 + 110 = 284\]
(iv) Maktalghis (triangular numbers), e.g., the numbers 1, 3, 6, 10, 15, 21, 28, 36, 45, which are the sum of the first one, first two, first three, first four and so on, natural numbers.

The Arabs also solved the famous problem of finding a square which, on the addition and subtraction of a given number, yields other squares. The extent of their knowledge of arithmetic can be gauged from the fact that al-Biruni was able to give the correct value of 10^14.1,2

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C

ALGEBRA

The ancient mathematicians, including the Greeks, considered the number to be a pure magnitude. It was only when al-Khwārizmi (d. 838/950) conceived of the number as a pure relation in the modern sense that the science of algebra could take its origin. The development of algebra is one of the greatest achievements of the Muslims, and it was cultivated so much that within two centuries of its creation it had reached considerable proportions. The symbolic process which it idealizes is still called "Algorithm" in modern mathematics. Al-Khwārizmi himself formulated and solved the algebraical equations of the first and second degree, and discovered his elegant geometrical method of finding the solution of such equations. He also recognized that the quadratic equation has two roots. Ibn-bīrāni (d. 1006) worked on geometry, especially on conic sections. His quadrature of the parabola was much simpler than that of Archimedes, in fact the simplest ever made before the invention of the integral calculus in the seventeenth century. Abu Kāmil Shūjā' al-Mišri developed the algebra of al-Khwārizmi, and determined the real roots of quadratic equations and their interpretations. Al-Khārizmī (d. c. 850/950) solved the cubic equation by employing the conic sections. Abu al-Wafā' al-Būzjāni (923/988-990-998) investigated and solved algebraic equations of the fourth degree of the type $x^4 = a$ and that of $x^4 + a^4 = b$. Al-Kāhi (fl. 978/998) investigated the solvability of algebraic equations. Abu Mahmud al-Khujandi (fl. 983/992) proved the so-called Fermat's problem for cubic powers, i.e., $x^3 + y^3 = z^3$, cannot be solved by rational numbers. Ibn al-Layth, who was a contemporary of al-Bīrāni, solved the problem which leads to the equation: $x^4 + 13.5x^2 + 5 = 10x^2$, and founded geometrical methods for solving cubic equations. Al-Bīrāni introduced the idea of "function," which, since the time of Leibniz (eleventh/seventeenth century), has become the most important concept in modern mathematics. Abu Bakr al-Karkhi, who is considered one of the greatest Arab mathematicians, wrote a book on algebra, called al-Fakhri, in which he developed approximate methods of finding square-roots; the theory of indices; the theory of surds; summation of series; equation of degree 2n; the theory of mathematical induction; and the theory of indeterminate quadratic equations. The next important figure is ibn al-Halīṣah (c. 354–431/c. 965–1039), who is recognized as the greatest physicist and expert on optics of the Middle Ages, and who solved the algebraic equation of the fourth degree by the method of intersection of the hyperbola and the circle.

Then came 'Umar al-Khayyām (c. 430–517/c. 1038–1123), who has recently become the most glamorous figure of the fifth/eighth century on account of his poetry, but who, according to Moritz Cantor, has better claim to immortality as a very great mathematician. He made what was for his time an uncommonly great progress by dealing systematically with equations of the cubic and higher orders and by classifying them into various groups according to their terms. He described thirteen different classes of cubic equations. He investigated the binomial expression for positive integral indices, i.e., in modern terminology, the expansion of $(1 + x)^n$, when $n$ is an integer. The next significant advance on this problem was made by Newton (eleventh/seventeenth century) when he proved the binomial theorem for any rational number. As stated by Cantor, Khayyām has a very exalted place in the history of algebra.

At about this time, Muslim scholars founded, developed, and perfected geometrical algebra, and could solve equations of the second, third, and fourth degree before the year 494/1100.

Moritz Cantor, who is by no means partial to the Muslims, remarks that "the Arabs of the year 494/1100 were uncommonly superior to the most learned Europeans of that time in the mathematical sciences." He goes on to relate the story that in the seventh/thirteenth century, Frederick II Hohenstaufen sent a special deputation to Mosul to ask Kamāl al-Dīn ibn Yūnus (d. 460/1062), the mathematician of a college later on called after him the Kamāl College, to solve some mathematical problems. Kamāl al-Dīn solved these problems for the Emperor. One of the questions solved by him was how to construct a square equivalent to a circular segment.

D

GEOMETRY

In the subject of geometry, the Arabs began by translating the Elements of Euclid and the Conics of Apollonius, thus preserving the work of these Greek masters for posterity. This task was satisfactorily accomplished in the early third/ninth century. Soon after this they launched on making fresh discoveries for themselves. The three brothers, Muhammad, Ahmad and Hasan, sons of Mūsā bin Shakir, may be regarded as pioneers in this field. They discovered a method of trisecting an angle by the geometry of motion, thus connecting geometry with mechanics. That this problem is not soluble by means of the ruler and compass alone, has been well known from the time of the Greek mathematicians. The brothers also worked on the mensuration of the sphere and on the ellipses.

In the fourth/fifth century, Abu al-Wafā', al-Kūhi, and others founded and successfully developed a branch of geometry which consists of problems leading

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5 Cantor, op. cit., p. 775.
6 Ibid., p. 778.
7 Ibid., p. 778.
8 Ibid.
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to algebraic equations of a degree higher than the second. Al-Khaki solved the problems of Archimedes and Apollonius by employing his new method. Abu Kamal al-Shafi’i, a mathematician, investigated geometrical figures of five and ten sides (pentagon and decagon) by algebraic methods. This co-ordination of geometry with algebra and the geometrical method of solving algebraic equations, like the application of geometry to algebra by Thabit bin Qurrah, a Syrian astronomer of the court of the Caliph Ma’tud, was the anticipation of Descartes’ great discovery of analytical geometry in the eighteenth/seventeenth century. Abu Sa’id al-Jazari “made a special study of the intersections of conic sections and circles. He replaced the old kinematical treatment of an angle by a purely geometrical solution (intersection of a circle and an equilateral hyperbola).”

Abu al-Wafa’i developed the method of solving geometrical problems with one opening of the compass, and of constructing a square equivalent to other squares. He made many valuable contributions to the theory of polyhedra, which is even now considered to be a very difficult subject. Ibn al-Haytham, known in Europe as Alhazen, also made many discoveries in geometry. His famous book on optics contains the following problem, known as Alhazen’s problem: from two points in the plane of a circle to draw lines meeting at a point of the circumference and making equal angles with the normal at that point. This problem leads to an equation of the fourth degree, and Ibn al-Haytham solved it by the aid of a hyperbola intersecting a circle.

The later Muslim mathematicians developed the geometry of the conic sections to some extent, but their greatest contribution was connected with the appeal of Euclid’s postulates. It is well known that in each science or logical system (such as the Euclidean geometry), the beginning is made with some fundamental concepts (like points and lines) and a few assertions or statements, called “postulates,” which are accepted without demonstration or proof, and on the basis of which further statements (called theorems) are established. Now it is recognized that some of Euclid’s postulates are quite self-evident. For instance, no one questions the validity of the statement that the whole is greater than a part or that equals added to equals result in equals. But the same cannot be said about Euclid’s parallel postulate. Fakhr al-Din Hāzī (d. 606/1209) made a preliminary critique of Euclid’s postulate, but it was Naṣr al-Dīn Tūsī (d. 673/1274), who, in the later half of the seventh/thirteenth century, recognized the weakness in Euclid’s theory of the parallels. In his efforts to improve the postulate, he realized the necessity of abandoning perceptual space. It was in the thirteenth/nineteenth century that such studies, continued by Gauss, Bolzoi, Lobachevsky, and Riemann,

resulted in the discovery and development of the various non-Euclidean geometries, culminating in the Theory of Relativity in our own time.

E

TRIGONOMETRY

Trigonometry, both plane and spherical, was developed to a great extent by the Arabs. Al-Khwarizmi himself compiled trigonometric tables, which contained not only the sine function, as done by his predecessors, but also that of the tangent, for the first time. These tables were translated into Latin by Abadal of Bath in 590/1190. Al-Battani (d. 929), known in Europe as Albategnus, devoted a whole chapter of his book on astronomy to the subject of trigonometry. He used sine regularly “with a clear consciousness of their superiority over the Greek chords.” The previous works contained only the full arc, but Al-Battani remarked that it was more advantageous to use the half arc. Cantor considers this “an advance in mathematics which cannot be appreciated highly enough.” Al-Battani completed the introduction of tangents and cotangents in trigonometry, and gave a table of cotangents by degrees. He knew the relation between the sides and angles of a spherical triangle which we express by the formula.

\[
cos a = \cos b \cos c + \sin b \sin c \cos a.
\]

Abu al-Wafai’s contribution to the development of trigonometry is well known. Most likely he was the first to show the generality of the sine theorem relative to the triangles. He introduced quite a new method of constructing sine tables, the value of sin 30° being correct to the eight decimal places. He knew relations equivalent to the present ones for \( \sin (a \pm b) \), and to

\[
2 \sin \frac{a}{2} = 1 - \cos a, \quad \sin a = 2 \sin \frac{a}{2} \cos \frac{a}{2}.
\]

He specially studied the tangent; drew up a table of tangents, introduced the secant and the cosecant in trigonometry, and knew those relations between the six trigonometric lines which are now often used to define them.

Al-Khwarizmi is considered to be the discoverer of the sine theorem relative to spherical triangles. This sine theorem displaced the theorem of Menelaus. Ibn Yūnus (d. 400/1006) made considerable contributions to trigonometry, and solved many problems of spherical astronomy by means of orthogonal

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11 Barton, op. cit., p. 485.
12 Ibid., p. 667.
13 Barton, op. cit., p. 711.
14 Ibid., p. 663.
15 Ibid., p. 663.
16 Cantor, op. cit., p. 717.
17 Barton, op. cit., p. 665.
18 Ibid., p. 667.
19 Ibid., p. 668.
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propositions. He discovered the first of those addition-subtraction formulae which were indispensable before the invention of logarithms, namely, the equivalent of

\[ \cos a \cos b = \frac{1}{2} [\cos(a - b) + \cos(a + b)] \]

He also gave a formula for the approximate value of \( \sin 1^\circ \).

Kūchīzīr ibn Labbān (fl. c. 821–830/971–1029) took an important part in the elaboration of trigonometry. For example, he continued the investigations on the tangent, and compiled comprehensive tables.81 Al-Zarqallī (fl. c. 430–480/1029–1087) explained the construction of the trigonometric tables, and compiled the Toledo Tables, which were translated into Latin by Gerard of Cremona and enjoyed much popularity.82

Al-Ḥasan al-Ḥarrārī (fl. c. 661/1262) introduced in 627/1229 the graphic method in trigonometry and prepared the tables of trigonometric functions. Naṣīr al-Dīn Tūsī wrote on plane and spherical trigonometry as a subject independent of astronomy.

Bābā al-Dīn (964–1032/1547–1622) gave in his book trigonometric methods for calculating heights and distances as well as for the determination of the breadth of a river.

F

ASTRONOMY

The Arabs claimed astronomy to be their own special subject. Indeed even at the beginning of Islam, they possessed sufficient astronomical knowledge to be able to use the position of stars in their wanderings and agriculture. But it was only in the second/seventh century that the scientific study of astronomy was begun.83 From this time up to the eighth-ninth/fourteenth-fifteenth century the contributions of Muslims to astronomy were so numerous that they can be dealt with adequately only in a separate volume. Here we summarize only the most important facts.

First of all let us take the observatories. Western historians have pointed out that before the advent of Islam, only one or two less well-known observatories existed in Alexandria, and even that was not doing much work. In the course of a few centuries, the Muslims erected innumerable well-equipped observatories all over their vast empire. Some of these observatories are as follows:

(i) The solar observatory built by al-Māmūn in Iraq in 214/829.
(ii) The Iṣḥāḥ observatory built by Abu Hannāfah al-Dīnāwī (d. 282/895).

81 Ibid., p. 717.
82 Ibid.
83 Ibid., p. 739.

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(iii) The Khwārizm observatory built by al-Bīrūnī.
(iv) The Baghdād observatory of Thābit ibn Qurrah.
(v) The Baghdād observatory built by Caliph al-Mustārghād, where the well-known astronomer Bādī made his observations.
(vi) The observatory erected by Ibn Sīnā.
(vii) The al-Raqāsh and Antikāsh (Antioch) observatories where al-Battānī made observations from 204/777 to 206/798.
(viii) The Bām Mūsā observatory at Baghdād.
(ix) The Sharaf al-Dīnāsh observatory where al-Sīgānī and al-Kuḥāī made their observations.
(x) The Tabīštā observatory where Abu Ikbar worked and made observations.
(xi) The Hūṣār observatory associated with the name of abu al-Wafā’.
(xii) The ibn Ḥārīam observatory built at Baghdād in 351–352/962–963.
(xiii) The Egyptian observatory where Ibn Yūsuf produced his famous almanac.
(xiv) The Māmūn observatory, associated with the name of Māmūn Batahī (d. 519/1125).
(xv) The Maragah observatory erected by Naṣīr al-Dīn Tūsī in 685/1289. It is said that several kinds of instruments were installed in this observatory, and that a library containing four hundred thousand volumes was attached to it.
(xvi) The observatory of Tūqī al-Dīnī.
(xvii) The Kūshī observatory.
(xviii) The Fīrūzābād observatory.
(xix) The Samarqand observatory erected by Sulṭān Ulugh Beg Mirza in 633/1240.

An account of these observatories lies scattered in various books, such as: Khudūz Tūrīghād al-ʿArab; Tamadunna-i Arab; Kūth al-Ḥajīn wa-al-Ābār; Sharākh Chaghami; Jāmī Tabāḍrī Kūhī; Muṣann al-Balābī; Ḥāfizī al-Qanū; Fudūd al-Wajībād; Rāsūl al-Ṣifī; Wolqūṣīt al-ʿArqī; Kūshī al-Jamūn.

Next to the observatories come the astronomical instruments; and the books on history record a large number of instruments constructed by the Arabs and other Muslim peoples. Work on astronomy of such magnitude could not be carried out with the rough instruments existing at the time. They had, therefore, to concentrate all their practical skill on devising elaborate instruments for making various observations. These have also been described in the books mentioned above. We shall confine ourselves to the enumeration and description of some important instruments.

(i) Lībānāt, built on a square base, served to measure the declination, latitude, and distances of the stars.
(ii) Hulqīkh Tūqīd (Meridian Circle), fixed in the plane of the meridian, and devised to determine the distances of the heavenly bodies.

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(iii) Zād al-A:')ār, constructed by Taqi al-Din, served as an alternative for the Meridian Circle which was useful during night as well as day.
(iv) Zād al-'Aqī (the Astrolabe) was one of the most important instruments. It consisted of two circles, one of which represented the ecliptic and the other the celestial meridian.
(v) Zād al-San')t al-?:专业人士 (Al-azimuth) consisted of a semi-circle and had the diameter of an equi-surfaced cylinder. Taqi al-Din has mentioned it in his work, to have been constructed by Muslim astronomers.
(vi) Zād al-?wā':Ita. It had three faces on one base and served to determine the altitude of the heavenly bodies.
(vii) Zād al-Ja?:Itt consisted of two faces and was used for the determination of the altitude.
(viii) Al-Mu'akb:ah bi al-Nā':Ita constructed by Taqi al-Din and used for determining the distance between two stars.
(ix) Tawāq al-Manqīt constructed by Ghikā'ī al-Din Ja'magh and used for determining the position of the stars, their latitudes, distance from the earth, and movement. It was also useful for obtaining data relating to lunar and solar eclipses.
(x) Zargānī constructed by Shā dit Ishaq ibn Yahya, generally known as al-Naqgī al-Andalusi (the Spanish painter). It was a very useful instrument for observing the movement of the heavenly bodies.
(xi) Zād al-Kharr:It constructed by Bad' al of the Astrolabe (Bad') al-Andalusi, as described by 'Abd al-Rahmān al-Sīfi.
(xii) Al-Mad al-?birna:It constructed by al-Khujandi and used for determining the latitudes.
(xiii) The several types of quadrants as described in Khulj al-Zarān.
(xiv) As-Surtīn Mijnah, the transit instrument described by Muhammad ibn Na'a: and Manṣūr ibn 'Ali.
(xv) Al-Ja?:It al-Qālit constructed of a semi-circle the circumference being divided equally.
(xvi) Sufi-Fe:kī, a sextant associated with the name of Fe:kī al-Danah Dai'lāni.

Now we shall describe briefly the investigations carried out by the Muslim astronomers. Although the regular observations and construction of astronomical instruments was started as early as the second/eighth century by Hurshā al-Fazā'iri (d. c. 180/796), the most brilliant period of Muslim astronomy commenced in the early part of the third/ninth century in the observatories constructed by the Caliph al-Mā'mūn (190–218/813–833). The observatory of Baghdad under Yahya bin abi Manṣūr (d. c. 216/831) made systematic observations of the heavenly bodies and found remarkably precise results for all the fundamental elements mentioned in Ptolemy’s Almagest, such as the obliquity of the ecliptic, the precession of the equinoxes, the length of the solar year. After recording these observations, Yahya compiled the celebrated “Tested Tables.” He was also the author of several works on astronomy.

Under the orders of al-Mā'mūn, the Muslim astronomers carried out one of the most delicate and difficult geodetic operations, the measuring of the arc of the meridian. The mean result gave 60?41 Arab miles as the length of a degree of meridian, which is a remarkably accurate value, for the Arabic mile is 6,673 ft. This value is equal to 306,842 ft., exceeding the real length of the degree between 38° and 36° latitudes by 2,877 ft.

Habash al-?ub was an astronomer under al-Mā'mūn and al-Mu'ta'jam; he compiled three astronomical tables, including the famous “Verified Tables.” Apoplos of the solar eclipse of 314/829, Habash gave the first instance of a determination of time by an altitude which was generally adopted by the astronomers.

Abu al-Aswā'ir was a famous maker of astronomical instruments. He took part in the degree measurement ordered by al-Mā'mūn, and wrote one of the earliest Arabic treatises on the astrolabe.

Al-Marwarrū:ī was one of those who took part in the solar observations made at Damascus in 217–218/832–833.

The three sons of Mūsā bin Ḫakir made regular observations in the observatories inBaghdad between 226/840 and 257/870.

Al-Farghā:ī was one of the most distinguished astronomers in the service of al-Mā'mūn and his successors. His famous work, Kāfib al-?arabil al-Samā'īyya wa Jawālī 'Ism al-Naujīn (Book on Celestial Motions and the Complete Science of the Stars), was translated into Latin in the sixteenth/seventeenth century. It exerted marked influence on European astronomy. He accepted Ptolemy’s theory and value of the precession but was of the view that it affected not only the stars but also the planets. He determined the diameter of the earth to be 6,500 miles, and found the greatest distances and also the diameters of the planets.

Al-Mahā:ī (d. between 261/874 and 271/884) made a series of observations of lunar and solar eclipses and planetary conjunctions during the years 292–322/883–906; these were later used by Ibn Yūnūs.

Al-Nairī (d. c. 310/922) compiled astronomical tables, made systematic observations, and wrote a book on atmospheric phenomena. He wrote a treatise on the spherical astrolabe which is very elaborate and is supposed to be the best Arabic work on the subject.

14 Ibid.
15 Rector, op. cit., p. 565.
16 Ibid., p. 566.
17 Ibid.
19 Rector, op. cit., p. 567.
20 Ibid., pp. 597–98.
21 Ibid., p. 599.
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Ṭḥabb ibn Qurrah published solar observations, explaining his methods. He revised the theory of the movement of the sun. To the eight Ptolemaic spheres, he made the addition of a ninth one (primum mobilis) to account for the imaginary trepidation of the equinoxes, which was, however, later found to be an erroneous theory. Al-Battānī was one of the greatest astronomers of the Middle Ages. He wrote many books but his main work, the famous De Numeris stellarum et matheis, exerted great influence in Europe up to the time of the Renaissance.

From 264/877 onwards he made astronomical observations of remarkable range and accuracy. His tables contain a catalogue of fixed stars for the year 267-68/880-81. He investigated the motion of the sun's apogee and found that its longitude had increased by 18°47' since the time of Ptolemy. This implied the discovery of the motion of the solar apsides, and of the slow variation in the equation of time. He determined many astronomical co-efficients with remarkable accuracy, and corrected the previous values of the precession of equinoxes and the obliquity of the ecliptic. He proved the possibility of the annual eclipses of the sun. He did not believe in the trepidation of the equinoxes, although the followers of Copernicus at a much later date did believe in it. Modern astronomy has shown that the Copernicans were wrong. He determined the moon's nodes and discovered the wobbling motion of the earth's orbit.

Ibn Amākir (abu Qāsim 'Abd Allah) together with his son ibn al-ʿAsaṣ 'Ali made many observations between 372/986 and 291/993 which were recorded by ibn Yaḥyā. They produced many astronomical tables, including the table of Mars according to Persian chronology. Abū al-Ḥasan discovered that the moon's distance from the sun is not constant as assumed by Ptolemy. Al-Kūhī was the leading astronomer working in 378/988 at the Khurasān al-Dalīlī observatory.

'Abd al-Raḥmān al-Sufi (291-370/903-986) was one of the most eminent Muslim astronomers. His chief work, Kitāb al-Kitāb al-Thūbīdī al-Masūṣ (Book of the Fixed Stars Illustrated), is regarded as one of the three masterpieces of Muslim observational astronomy, the other two being one by ibn Yūnūs and a work prepared for Ulugh Beg.

Ibn al-Sam (d. 375/985) has been praised for the accuracy of his observations; his tables continued to be very popular for at least two centuries. He determined the stellar motion by observing that the star traverse on

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degree in seventy solar years. He also determined the latitude and longitude of many stars, and measured the greatest declination of the planet Mercury. He found that the earth is spherical and may, therefore, be supposed to be inhabited everywhere. He discovered the satellites of Jupiter, discussed the motion of the sun's spots, and determined the eccentric orbit of the comets. Abū al-Wafāʾ al-Bīrūnī determined accurately the obliquity of the ecliptic in 344/955, and calculated the variation in the moon's motion. There is a difference of opinion about his discovery of the third liberation in the moon's motion. Some of the older writers believed that he discovered the third liberation and that Tycho Brahe rediscovered it in the tenth and sixteenth century. But Sarton remarks that al-Wafāʾ did not discover this variation, but simply spoke of the second part of the ejection, which is essentially different from the variation discovered by Tycho Brahe.

Al-Khujandi made astronomical observations, including a determination of the obliquity of the ecliptic, at Bāyāy, in 394/994. Maslamah ibn Ahmad al-Maʾṣūf (d. c. 398/1007) edited and corrected the astronomical tables of al-Khwārizmi replacing the Persian by the Arabic chronology. He wrote a treatise on the astrolabe and a commentary on Ptolemy's Planisphaerium both of which were later translated into Latin.

Ibn Yūnus has been described by Sarton as the greatest Muslim astronomer. A well-equipped observatory in Cairo enabled him to prepare improved astronomical tables, called al-Eṣʿāʾ al-Kallār al-Ḥikmāt, completed in 396/1007. They describe observations of eclipses and conjunctions, old and new, and improved value of astronomical constants (obliquity of the ecliptic 23°35'; longitude of the sun's apogee 86°10'; solar parallax reduced from 2.5 to 2'. precession of the equinoxes 51.2' per annum), and give an account of the geocentric measurements made under al-Maʾṣūf's order. He is specially noted for his method of longitude determination. As time difference is equivalent to longitude difference, the determination of local time at the same instant at two stations widely separated in longitude is sufficient. But there were no telegraphs or radio signals to give simultaneously. Ibn Yūnus proposed and used a signal from the moon—the first contact of a lunar eclipse. In this way he corrected many errors in longitudes in Ptolemy's geography.

41 Ḥabib al-Qārimi, p. 23.
42 Ḥabīb al-Qārimi, p. 248.
43 Ghulam Aḥmad Amin, Jamiʿ al-Bahūd al-Ḥusn, p. 596.
44 Ibid., p. 668.
46 Khojaev Tūrāji al-ʿArab, 243.
47 Sarton, op. cit., p. 666.
48 Ibid., p. 668.
49 Ibid.
50 Ibid., p. 716.
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Al-Birūnī is regarded by Western historians of science as "one of the greatest scientists of all times whose critical spirit, tolerance, love of truth, and intellectual courage were almost without parallel in medieval times." He made accurate determination of latitudes and longitudes and also other geodetic measurements. He discussed in his book Qādīn al-Maṣūdī for the first time the question that the earth rotates around its axis. The translation of the relevant Arabic passages is as follows: "When a thing falls from a height, it does not coincide with the perpendicular line of its descent, but inclines a little, and falls making different angles. When a piece of earth separates from it and falls, it has two kinds of motion: one is the circular motion which it owes from the rotation of the earth, and the other is straight which it acquires in falling directly to the centre of the earth. If it had only the straight motion, it would have fallen to the west of its perpendicular position. But since both of them exist at one and the same time, it falls neither to the west nor in the perpendicular direction, but a little to the east." This book of Al-Biruni, Viz. al-Qādīn al-Maṣūdī, was written in 425/1030, and gave the true explanation of the rising and setting of the heavenly bodies as being due to the rotation of the earth, thus pointing to the error in the geocentric conception of the solar system. The heliocentric doctrine was not entirely unknown to the Arabs, who knew that the earth revolved round the sun and that the orbits of the planets were elliptic. It should be noted that Copernicus gave the scientific formulation and detailed working out of the heliocentric theory some three centuries later.

Al-Zarqālī was "the best observer of his time. He invented an improved astrolabe called Saffihah; his description of it was translated into Latin, Hebrew, and many vernaculars. He was the first to prove explicitly the motion of the solar apogee with reference to the stars; according to his measurements it amounted to 12°4′ per year (the real value being 11°8′)." He edited the planetary tables called the "Toledan Tables." Umār Khāṣyām was called to the new observatory of Rayy in 467/1074 by Sulṭān Malik šah Jalāl al-Dīn Saljūq to reform the old calendar. Moritz Cantor remarks that the calendar prepared by 'Umār Khāṣyām, called al-Tūrīkh al-Jalālī, was more accurate than any other proposed before or after his time. Its date was 10th Ramadān 471, i.e., 16th March 1079. The modern interpretation of Khāṣyām's calendar is that eight intercalary days should be introduced in thirty-three years, resulting in an error of one day in about 3,000 years. The Gregorian calendar leads to an error of one day in 3,300 years. Ghīnī Khān erected a magnificent observatory at Maraghah near Tabriz far surpassing any built by his predecessors. Nasīr al-Dīn Tūsī was the greatest genius of this institution. He was quite original and independent, and criticized

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Ptolemy quite severely, "paying the way for the overthrow of the geocentric system."

Ulugh Beg, grandson of Timūr, established an observatory at Samarqand, Turkestan, in 192/1430, which was best equipped. A great work produced at this observatory was an independent star catalogue, known as the "Ulugh Beg Tables," based entirely upon new observations, the first in about sixteen hundred years, i.e., since the time of Hipparchus, second century B.C. The positions were given to the nearest minute of arc, and attained a high degree of precision for that period. Instruments used in this observatory are considered the best made up to that time. It is said that his quadrant was so large that its diameter was equal to the height of the St. Sophia Church in Constantinople. This work on astronomy is regarded as one of the best books of the Muslim astronomers. It was written in 1441/1447, and from it one can have a fair account of the knowledge possessed by the Muslims in the ninth/tenth century. The first part deals with the general principles of astronomy. The latter part contains the practical methods of calculating the lunar and solar eclipses and the construction of the tables and their applications; a list of the stars; the motion of the sun, the moon, and the planets; and the terrestrial latitudes and longitudes of the big cities of the world. The Mughuls inherited their fondness for astronomy from Ulugh Beg. Farīdīhān remarks that Humāyūn was a keen astronomer and spent a good deal of time in its pursuit. An observatory was founded in Delhi under the orders of Muhammad Shāh in 1357/1724, which was in the charge of the well-known mathematician Mirza Khair Allah. By this time the West had made great progress in astronomy as in other branches of knowledge, and therefore a commission consisting of the ablest men of the time was sent to Europe to study the new methods followed there and new results obtained through the then latest researches. The commission brought back with it some telescopes and other instruments and a few books prepared in Europe. The King of Portugal also deputed a European astronomer to go to Delhi with the commission. But when his data were checked at the Delhi observatory, local people detected errors and made corrections in his tables and calculations of the lunar and solar eclipses. This is ascribed to the fact that the instruments made in Europe at the time were of a smaller size than those available in the Delhi observatory.

The Nisfīntīyah observatory was erected at Hyderabad Deccan in the thirteenth/nineteenth century, and was the biggest institution of its kind in the East. It contained a sixteen-inch refracting telescope, a transit instrument, a Meridian circle, and a good deal of other equipment essential for a modern
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observatory. Its unique position was recognized by international organizations, and it had an important share in the preparation of the International Catalogue of Stars. After the establishment of the Osmania University, it became a constituent unit of that University.

The influence of the Muslims in this field is traceable from the many Arabic names and words that have become an integral part of the astronomical sciences. A long list of such words can be compiled, but it would be sufficient to mention a few: almanac (al-ma’tāz), almacantar (al-maqta’rah), nadir (nādir), zenith (ran al-rūs), algal (al-qāhir), aitair (al-istair), ibn bahr (al-ibn bahr), fahal (fat al-hal), denah (dhahab), vega (wāqī), and the various names of Muslim astronomers given to the craters of the moon.43


BIBLIOGRAPHY


Chapter LXIV

PHYSICS AND MINERALOGY

The Muslims contributed enormously to exact sciences such as mathematics, astronomy, physics, chemistry, botany, and zoology since they had succeeded in acquiring the knowledge of the sciences which had developed before the advent of Islam.

Abu Yusuf Ya’qub ibn Ishaq al-Khāki4 is the first Muslim scientist-philosopher. His pure Arabian descent earned him the title “The Philosopher of the Arabs.” Indeed, he was the first and last example of an Aristotelian student in the Eastern Caliphate who sprang from the Arabian stock. His principal work on geometrical and physiological optics based on the optics of Euclid in Theon’s recension was widely used both in the East and the West until it was superseded by the greater work of Ibn al-Haitham. He was the first Muslim to write in Arabic a book on music in which he designed a notation for the pitch of notes. Al-Kindi’s three or four treatises on the theory of music are the earliest extant works in Arabic showing the influence of Greek writings on that subject. Of al-Kindi’s writings more have survived in Latin translations than in the Arabic original.4

An observatory was opened by the three sons of Maim ibn Shahin (326–327/820–870) in their house at Bagdad. The Buwayhid Sultan al-Hasan al-Dinawarī (372–378/982–989) instituted another in his palace at Bagdad where ‘Abd al-Rahman al-Siṣṭī (d. 375/986), Ahmad al-Maṣūṣī (d. 380/990), and Abu al-Walī (d. 387/997) carried out their astronomical observations. At the Court of another Buwayhid, Ebn al-Dinawarī (392–396/992–996) of al-Rayy, nourished Abu Ja’far al-Khāzīn of Khurasan who ascended the obliquity of the ecliptic and solved a problem in Archimedes which led to the discovery of a cubic equation. Other astronomers made a systematic study of the heavens in Shiraz, Nayshabur, and Samarkand. Ibn Mīnā published a work on the balance.

‘Uṯrīd ibn Muhammad al-Ḫaṣīb wrote a book on lapidary which is reckoned among the oldest Arabic works on this subject; Abu Zakārīya al-Rāzī quoted from ‘Uṯrīd in his famous book al-Ḫaṣīb. Al-Rāzī the Iranian was one of the greatest medical men of the Middle Ages. He was an expert chemist and physicist.

Al-Hakim the Fatimid was personally interested in astronomical calculations. He built on the Macquttan an observatory to which he used to ride before dawn. The intellectual lights of his Court were ‘Ali ibn Yūsim (d. 400/1009), the greatest astronomer Egypt has ever produced, and Abu ‘Ali al-Hațīm (Latin Alhazen), the principal Muslim physicist and student of optics. The latter was undoubtedly the foremost physicist of the Middle Ages. His researches into geometrical and physiological optics were considered to be the most important and useful up to the time of Renaissance. His explanation of the vision and functions of the eye was far in advance of the ideas of the ancients. The chief work for which he is noted is one on optics, Kitāb al-Manāzir, of which the original is lost but which was translated into Latin in the sixteenth/seventeenth century. Almost all the medieval writers on optics in the West based their works on Ibn Hațīm’s Optics Theorems.

In this work he opposed the theory of Euclid and Ptolemy that the eye sends out visual rays to the object of vision, and presented experiments for testing the angles of incidence and refraction. In certain experiments he approached the theoretical discovery of magnifying lenses which were manufactured in Europe centuries later.4

4 Hād., p. 376.
4 Hād., p. 629.

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