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Chapter LXIII

MATHEMATICS AND ASTRONOMY

A

INTRODUCTION

It is generally recognized that human knowledge took its organized and systematic form with the Greeks. It is equally well known that the Greeks inherited a considerable body of knowledge from their Eastern predecessors, especially the Egyptians, Babylonians, Chinese, and Indians.

The histories of science and culture, written by some Western writers, however, show a gap between the period of the Greeks and the Renaissance. They give the impression that the history of science was blank for nearly one thousand years, and scientific knowledge made a sudden leap, taking a millennium in its stride. These histories ignore the fact that the intervening ages from the first/seventh to the eighth/fourteenth century constituted the era of the Arab and other Muslim peoples.

The latest researches of Muslim and non-Muslim scholars are bringing to light the work of the Muslims in the various branches of knowledge throughout the Middle Ages. These researches are, however, scattered in various journals and books which are not easily accessible to the average educated person. Two good works of reference published are the *Encyclopaedia of Islam* and George Sarton's *Introduction to the History of Science.* On a thorough study of the information available on the subject, one is struck by the magnitude as well as importance of the contributions made by the Muslims to the various branches of science, especially to mathematics and astronomy.

The magnitude of these achievements is so vast that it is giving rise to another tendency among the historians of science. It is incomprehensible to them that the Arabs who were so backward and ignorant in the centuries preceding the advent of Islam could have become so enlightened and scholarly in such a short time after adopting the new faith. One of the great exponents of this line of thought is Moritz Cantor who has written an encyclopedic history of mathematics in the German language. The chapter on the Arabs in Cantor's book begins as follows:

> "That a people who for centuries together were closed to all the cultural influences from their neighbours, who themselves did not influence others

during all this time, who then all of a sudden imposed their faith, their laws, and their language on other nations to an extent which has no parallel in history—all this is such an extraordinary phenomenon that it is worthwhile to investigate its causes. At the same time we can be sure that this sudden outburst of intellectual maturity could not have originated of itself."

Labouring under this fixed idea, Cantor proceeds to attribute almost everything done by the Muslim scholars to the Greeks and other nations. We must confess that this kind of argument introduces an extremely dangerous principle in historical research, and can be employed only by one who is predisposed to demolish an exalted and established reputation. If Cantor had really investigated the cause of the "sudden outburst of intellectual maturity" of the Arabs, he would have realized that it was primarily due to the revolution caused by Islam in the whole outlook of the people. We have elsewhere described the attitude of Islam towards knowledge.[1] By making it incumbent upon the believer to acquire knowledge and by enjoining upon him to observe and to think for himself, Islam created an unbounded enthusiasm for acquiring knowledge amongst its followers. The result of this revolution can be best described in the words of Florian Cajori, who says in his *History of Mathematical Notation*: "The Arabs present an extraordinary spectacle in the history of civilization. Unknown, ignorant, and disunited tribes of the Arabian Peninsula, untrained in government and war, are, in the course of ten years, fused by the furnace-blast of religious enthusiasm into a powerful nation, which in one century extends its dominion from India across northern Africa to Spain. A hundred years after this grand march of conquest, we see them assume the leadership of intellectual pursuits; the Muslims become the great scholars of their time."

It is under this stimulus of the Islamic injunction for acquiring more and more knowledge that the Arabs and other Muslim peoples turned to the learning of the various branches of knowledge, preserving and improving upon the heritage left by preceding civilizations and enriching every subject to which they turned their attention. In the following pages we give an account of their contribution in the domain of mathematics and astronomy. It may be pointed out that this is only a brief chapter in the general history of Muslim philosophy. The account will, therefore, be of a descriptive nature, shorn of all technicalities and confined to some of the fundamental ideas put forward by the Muslim peoples in the fields of arithmetic, algebra, geometry, trigonometry, and astronomy. It is neither possible nor desirable to give here an exhaustive account of the work done by each and every Muslim scholar. We have restricted ourselves to important contributions of the prominent Muslim mathematicians and astronomers.

[1] Vol. I, Chap. VIII.

B

ARITHMETIC

The Arabs started work on arithmetic in the second/eighth century. Their first task in this field was to systematize the use of the Hindu numerals which are now permanently associated with their names. Obviously, this was an immense advance on the method of depicting numbers by the letters of the alphabet which was universal up to that time and which prevailed in Europe even during the Middle Ages. The rapid development in mathematics in the subsequent ages could not have taken place without the use of numerals, particularly zero without which all but the simplest calculations become too cumbersome and unmanageable. The zero was mentioned for the first time in the arithmetical work of al-Khwārizmi written early in the third/ninth century. The Arabs did not confine their arithmetic to integers only, but also contributed a great deal to the rational numbers consisting of fractions. This was the first extension of the domain of numbers, which, in its logical development, led to the real, complex, and hyper-complex numbers constituting a great part of modern analysis and algebra. They also developed the principle of error which is employed in solving algebraic problems arithmetically. Al-Bīrūni (363–432/973–1040), ibn Sīna (370–428/980–1037), ibn al-Samḥ (d. 427/1035), Muḥammad ibn Ḥusain al-Karkhi (d. 410/1019 or 420/1029), abu Sa'īd al-Sijzi (c. 340–c. 415/c. 951–c. 1024) are some of the arithmeticians who worked on the higher theory of numbers and developed the various types of numbers, such as:

(i) *Tāmm* (perfect numbers), i.e., those which are equal to the sum of their divisors, e.g., $6 = 1 + 2 + 3$.

(ii) *Muta'ādilān* (equivalents), i.e., two numbers, the sum of the divisors of which is the same, e.g., 39 and 55: $1 + 3 + 13 = 1 + 5 + 11$.

(iii) *Mutaḥābbān* (amicable numbers), i.e., two such numbers in which the sum of the divisors of one equal the other, e.g., 220 and 284:
$1 + 2 + 4 + 71 + 142 = 220$
$1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$.

(iv) *Muthallathāt* (triangular numbers), e.g., the numbers 1, 3, 6, 10, 15, 21, 28, 36, 45, which are the sum of the first one, first two, first three, first four and so on, natural numbers.[2]

The Arabs also solved the famous problem of finding a square which, on the addition and subtraction of a given number, yields other squares.[3]

The extent of their knowledge of arithmetic can be gauged from the fact that al-Bīrūni was able to give the correct value of $16^{16}-1$.[4]

[2] *Encyclopaedia of Islam*, Vol. I, p. 124.
[3] Moritz Cantor, *Geschichte der Mathematik*, Vol. I, p. 752.
[4] George Sarton, *Introduction to the History of Science*, Vol. I, p. 707.

C

ALGEBRA

The ancient mathematicians, including the Greeks, considered the number to be a pure magnitude. It was only when al-Khwārizmi (d. 236/850) conceived of the number as a pure relation in the modern sense that the science of algebra could take its origin. The development of algebra is one of the greatest achievements of the Muslims, and it was cultivated so much that within two centuries of its creation it had reached considerable proportions. The symbolical process which it idealizes is still called "Algorithm" in modern mathematics. Al-Khwārizmi himself formulated and solved the algebraic equations of the first and second degree, and discovered his elegant geometrical method of finding the solution of such equations. He also recognized that the quadratic equation has two roots. Ibrāhīm ibn Sinān (296–335/908–946) worked on geometry, especially on conic sections. His quadrature of the parabola was much simpler than that of Archimedes, in fact the simplest ever made before the invention of the integral calculus in the eleventh/seventeenth century.[5] Abu Kāmil Shujā' al-Miṣri developed the algebra of al-Khwārizmi, and determined the real roots of quadratic equations and their interpretations. Al-Khāzin (d. c. 350/961) solved the cubic equation by employing the conic sections.[6] Abu al-Wafā' (al-Būzjāni) (329–388/940–998) investigated and solved algebraic equations of the fourth degree of the type $x^4 = a$, and that of $x^4 + ax^3 = b$. Al-Kuhi (fl. c. 378/988) investigated the solvability of algebraic equations. Abu Maḥmūd al-Khujandi (fl. 382/992) proved that the so-called Fermat's problem for cubic powers, i.e., $x^3 + y^3 = z^3$, cannot be solved by rational numbers. Ibn al-Laith, who was a contemporary of al-Bīrūni, solved the problem which leads to the equation: $x^3 + 13.5x + 5 = 10x^2$, and founded geometrical methods for solving cubic equations. Al-Bīrūni introduced the idea of "function," which, since the time of Leibniz (eleventh/seventeenth century), has become the most important concept in modern mathematics. Abu Bakr al-Karkhi, who is considered one of the greatest Arab mathematicians, wrote a book on algebra, called *al-Fakhri*, in which he developed approximate methods of finding square-roots; the theory of indices; the theory of surds; summation of series; equation of degree 2n; the theory of mathematical induction; and the theory of indeterminate quadratic equations.

The next important figure is ibn al-Haitham (c. 354–431/c. 965–1039), who is recognized as the greatest physicist and expert on optics of the Middle Ages, and who solved the algebraic equation of the fourth degree by the method of intersection of the hyperbola and the circle.

Then came 'Umar al-Khayyām (c. 430–517/c. 1038–1123), who has recently become the most glamorous figure of the fifth/eleventh century on account

[5] *Ibid.*, p. 632.

[6] Heinrich Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke.*

of his poetry, but who, according to Moritz Cantor, has better claim to immortality as a very great mathematician. He made what was for his time an uncommonly great progress by dealing systematically with equations of the cubic and higher orders and by classifying them into various groups according to their terms.[7] He described thirteen different classes of cubic equations. He investigated the binomial expression for positive integral indices, i.e., in modern terminology, the expansion of $(1 + x)^n$, when n is an integer. The next significant advance on this problem was made by Newton (eleventh/seventeenth century) when he proved the binomial theorem for any rational number. As stated by Cantor, Khayyām has a very exalted place in the history of algebra.[8]

At about this time, Muslim scholars founded, developed, and perfected geometrical algebra, and could solve equations of the second, third, and fourth degree before the year 494/1100.

Moritz Cantor, who is by no means partial to the Muslims, remarks that "the Arabs of the year 494/1100 were uncommonly superior to the most learned Europeans of that time in the mathematical sciences."[9] He goes on to relate the story that in the seventh/thirteenth century, Frederick II Hohenstaufen sent a special deputation to Mosul to ask Kamāl al-Dīn ibn Yūnus (d. 640/1242), the mathematician of a college later on called after him the Kamalic College, to solve some mathematical problems. Kamāl al-Dīn solved these problems for the Emperor.[10] One of the questions solved by him was how to construct a square equivalent to a circular segment.

D

GEOMETRY

In the subject of geometry, the Arabs began by translating the *Elements* of Euclid and the *Conics* of Apollonius, thus preserving the work of these Greek masters for posterity. This task was satisfactorily accomplished in the early third/ninth century. Soon after this they launched on making fresh discoveries for themselves. The three brothers, Muḥammad, Aḥmad and Ḥasan, sons of Mūsa bin Shākir, may be regarded as pioneers in this field. They discovered a method of trisecting an angle by the geometry of motion, thus connecting geometry with mechanics. That this problem is not solvable by means of the ruler and compass alone, has been well known from the time of the Greek mathematicians. The brothers also worked on the mensuration of the sphere and on the ellipse.

In the fourth/tenth century, abu al-Wafā', al-Kuhi, and others founded and successfully developed a branch of geometry which consists of problems leading

[7] Cantor, *op. cit.*, p. 775.

[8] *Ibid.*, p. 776.

[9] *Ibid.*, p. 778.

[10] *Ibid.*

to algebraic equations of a degree higher than the second. Al-Kuhi solved the problems of Archimedes and Apollonius by employing this new method. Abu Kāmil Shujā' al-Ḥāsib al-Miṣri investigated geometrical figures of five and ten sides (pentagon and decagon) by algebraic methods. This co-ordination of geometry with algebra and the geometrical method of solving algebraic equations, like the application of geometry to algebra by Thābit bin Qurrah, a Ṣābian astronomer of the court of the Caliph Mu'taḍid, was the anticipation of Descartes' great discovery of analytical geometry in the eleventh/seventeenth century. Abu Sa'īd al-Sijzi "made a special study of the intersections of conic sections and circles. He replaced the old kinematical trisection of an angle by a purely geometrical solution (intersection of a circle and an equilateral hyperbola)."[11]

Abu al-Wafā' developed the method of solving geometrical problems with one opening of the compass, and of constructing a square equivalent to other squares. He made many valuable contributions to the theory of polyhedra, which is even now considered to be a very difficult subject.[12]

Ibn al-Haitham, known in Europe as Alhazen, also made many discoveries in geometry. His famous book on optics contains the following problem, known as Alhazen's problem: from two points in the plane of a circle to draw lines meeting at a point of the circumference and making equal angles with the normal at that point. This problem leads to an equation of the fourth degree, and ibn al-Haitham solved it by the aid of a hyperbola intersecting a circle.[13]

The later Muslim mathematicians developed the geometry of the conic sections to some extent, but their great contribution was connected with the appraisal of Euclid's postulates. It is well known that in each science or logical system (such as the Euclidean geometry), the beginning is made with some fundamental concepts (like points and lines) and a few assertions or statements, called "postulates," which are accepted without demonstration or proof, and on the basis of which further statements (called theorems) are established. Now it is recognized that some of Euclid's postulates are quite self-evident. For instance, no one questions the validity of the statement that the whole is greater than a part or that equals added to equals result in equals. But the same cannot be said about Euclid's parallel postulate. Fakhr al-Dīn Rāzi (d. 606/1209) made a preliminary critique of Euclid's postulate, but it was Naṣīr al-Dīn Ṭūsi (d. 673/1274), who, in the later half of the seventh/thirteenth century, recognized the weakness in Euclid's theory of the parallels. In his efforts to improve the postulate, he realized the necessity of abandoning perceptual space. It was in the thirteenth/nineteenth century that such studies, continued by Gauss, Bolyai, Lobachevsky, and Riemann,

[11] Sarton, *op. cit.*, p. 665.
[12] *Ibid.*, p. 667.
[13] *Ibid.*, p. 721.

resulted in the discovery and development of the various non-Euclidean geometries, culminating in the Theory of Relativity in our own time.

E

TRIGONOMETRY

Trigonometry, both plane and spherical, was developed to a great extent by the Arabs. Al-Khwārizmi himself compiled trigonometric tables, which contained not only the sine function, as done by his predecessors, but also that of the tangent, for the first time. These tables were translated into Latin by Adelard of Bath in 520/1126.[14]

Al-Battāni (d. 317/929), known in Europe as Albategnius, devoted a whole chapter of his book on astronomy to the subject of trigonometry. He used sines regularly "with a clear consciousness of their superiority over the Greek chords."[15] The previous works contained only the full arc, but al-Battāni remarked that it was more advantageous to use the half arc. Cantor considers this "an advance in mathematics which cannot be appreciated highly enough."[16] Al-Battāni completed the introduction of tangents and cotangents in trigonometry, and gave a table of cotangents by degrees. He knew the relation between the sides and angles of a spherical triangle which we express by the formula:[17]

$$\cos a = \cos b \cos c + \sin b \sin c \cos a.$$

Abu al-Wafā's contribution to the development of trigonometry is well known. Most likely he was the first to show the generality of the sine theorem relative to the triangles. He introduced quite a new method of constructing sine tables, the value of sin 30′ being correct to the eight decimal places. He knew relations equivalent to the present ones for sin $(a \pm b)$, and to

$$2 \sin^2 \frac{a}{2} = 1 - \cos a,$$
$$\sin a = 2 \sin \frac{a}{2} \cos \frac{a}{2}.$$

He specially studied the tangent; drew up a table of tangents, introduced the secant and the cosecant in trigonometry, and knew those relations between the six trigonometric lines which are now often used to define them.[18]

Al-Khujandi is considered to be the discoverer of the sine theorem relative to spherical triangles. This sine theorem displaced the theorem of Menelaos.[19]

Ibn Yūnus (d. 400/1009) made considerable contributions to trigonometry, and solved many problems of spherical astronomy by means of orthogonal

[14] *Ibid.*, p. 563.
[15] *Ibid.*, p. 603.
[16] Cantor, *op. cit.*, p. 737.
[17] Sarton, *op. cit.*, p. 603.
[18] *Ibid.*, p. 667.
[19] *Ibid.*, p. 668.

projections. He discovered the first of those addition-subtraction formulae which were indispensable before the invention of logarithms, namely, the equivalent of

$$\cos a \cos b = \tfrac{1}{2}[\cos(a-b) + \cos(a+b)]$$

He also gave a formula for the approximate value of sin 1°.[20]

Kūshyār ibn Labbān (fl. c. 361–420/971–1029) took an important part in the elaboration of trigonometry. For example, he continued the investigations on the tangent, and compiled comprehensive tables.[21]

Al-Zarqāli (fl. c. 420–480/1029–1087) explained the construction of the trigonometric tables, and compiled the Toledan Tables, which were translated into Latin by Gerard of Cremona and enjoyed much popularity.[22]

Al-Ḥasan al-Marrākushi (fl. c. 661/1262) introduced in 627/1229 the graphic method in trigonometry and prepared the tables of trigonometric functions.

Naṣir al-Dīn Ṭūsi wrote on plane and spherical trigonometry as a subject independent of astronomy.

Bahā' al-Dīn (954–1032/1547–1622) gave in his book trigonometric methods for calculating heights and distances as well as for the determination of the breadth of a river.

F

ASTRONOMY

The Arabs claimed astronomy to be their own special subject. Indeed even at the beginning of Islam, they possessed sufficient astronomical knowledge to be able to use the position of stars in their wanderings and agriculture. But it was only in the second/eighth century that the scientific study of astronomy was begun.[23] From this time up to the eighth-ninth/fourteenth-fifteenth century the contributions of Muslims to astronomy were so numerous that they can be dealt with adequately only in a separate volume. Here we summarize only some of the most important facts.

First of all let us take the observatories. Western historians have pointed out that before the advent of Islam, only one more or less well-known observatory existed in Alexandria, and even that was not doing much work. In the course of a few centuries, the Muslims erected innumerable well-equipped observatories all over their vast empire. Some of these observatories are as follows:

(i) The solar observatory built by al-Māmūn in Iraq in 214/829.
(ii) The Iṣpahān observatory built by abu Ḥanīfah al-Dīnāwari (d. 282/895).

[20] *Ibid.*, p. 717.
[21] *Ibid.*
[22] *Ibid.*, p. 759.
[23] *Encyclopaedia of Islam*, Vol. I, p. 498.

(iii) The Khwārizm observatory built by al-Bīrūni.
(iv) The Baghdād observatory of Thābit ibn Qurrah.
(v) The Baghdād observatory built by Caliph al-Mustarshid, where the well-known astronomer Badī' made his observations.
(vi) The observatory erected by ibn Sīna.
(vii) The al-Raqqah and Anṭākiyah (Antioch) observatories where al-Battāni made observations from 264/877 to 306/918.
(viii) The banu Mūsa observatory at Baghdād.
(ix) The Sharaf al-Daulah observatory where al-Ṣāghāni and al-Kuhi made their observations.
(x) The Ṭabiṭala observatory where abu Isḥāq worked and made observations.
(xi) The Būzjān observatory associated with the name of abu al-Wafā'.
(xii) The ibn A'lam observatory built at Baghdād in 351–352/962–963.
(xiii) The Egyptian observatory where ibn Yūnus produced his famous almanac.
(xiv) The Māmūni observatory, associated with the name of Māmūn Bataihi (d. 519/1125).
(xv) The Marāghah observatory erected by Naṣir al-Dīn Ṭūsi in 658/1259. It is said that several kinds of instruments were installed in this observatory, and that a library containing four hundred thousand volumes was attached to it.
(xvi) The observatory of Taqi al-Dīn.
(xvii) The Kashmīr observatory.
(xviii) The Fīrūzshāhi observatory.
(xix) The Samarqand observatory erected by Sulṭān Ulugh Beg Mirza in 823/1420.

An account of these observatories lies scattered in various books, such as: *Khulāṣah Tārīkh al-'Arab*; *Tamaddun-i 'Arab*; *Kitāb al-Khiṭaṭ w-al-Āthār*; *Sharh Chaghmani*; *Jāmi' Bahādur Khāni*; *Mu'jam al-Buldān*; *Iktifā' al-Qunū'*; *Fuwāt al-Wajnāt*; *Rauḍat al-Ṣafā'*; *Wafayāt al-A'yān*; *Kashf al-Ẓunūn*.

Next to the observatories come the astronomical instruments; and the books on history record a large number of instruments constructed by the Arabs and other Muslim peoples. Work on astronomy of such magnitude could not be carried out with the rough instruments existing at the time. They had, therefore, to concentrate all their practical skill on devising elaborate instruments for making various observations. These have also been described in the books mentioned above. We shall confine ourselves to the enumeration and description of some important instruments.

(i) *Libnah*, built on a square base, served to measure the declination, latitude, and distances of the stars.
(ii) *Ḥalqah I'tidāl* (Meridian Circle), fixed in the plane of the meridian, and devised to determine the distances of the heavenly bodies.

(iii) *Dhāt al-Autār*, constructed by Taqi al-Dīn, served as an alternative for the Meridian Circle which was useful during night as well as day.

(iv) *Dhāt al-'Alq* (the Astrolabe) was one of the most important instruments. It consisted of two circles, one of which represented the ecliptic and other the celestial meridian.

(v) *Dhāt al-Samt w-al-Irtifā'* (Alt-azimuth) consisted of a semi-circle and had the diameter of an equi-surfaced cylinder. Taqi al-Dīn has mentioned it in his work, to have been constructed by Muslim astronomers.

(vi) *Dhāt al-Shu'batain.* It had three faces on one base and served to determine the altitude of the heavenly bodies.

(vii) *Dhāt al-Jaib* consisted of two faces and was used for the determination of the altitude.

(viii) *Al-Mushabbah bi al-Nāṭiq* constructed by Taqi al-Dīn and used for determining the distance between two stars.

(ix) *Ṭabaq al-Manāṭiq* constructed by Ghiyāth al-Dīn Jamshīd and used for determining the position of the stars, their latitudes, distance from the earth, and movement. It was also useful for obtaining data relating to lunar and solar eclipses.

(x) *Zarqālah* constructed by Shaikh Isḥāq ibn Yaḥya, generally known as *al-Naqqāsh al-Andalusi* (the Spanish painter). It was a very useful instrument for observing the movement of the heavenly bodies.

(xi) *Dhāt al-Kursi* constructed by Badī' of the Astrolabe (Badī' al-Asṭurlābi), as described by 'Abd al-Raḥmān al-Ṣūfi.

(xii) *Al-Ālat al-Shāmilah* constructed by al-Khujandi and used for determining the latitudes.

(xiii) The several types of quadrants as described in *Kashf al-Ẓunūn.*

(xiv) *Asṭurlāb Sarṭāni Mijnaḥ*, the transit instrument described by Muḥammad ibn Naṣr and Manṣūr ibn 'Ali.

(xv) *Al-Jaib al-Ghā'ib* consisting of a semi-circle the circumference being divided equally.

(xvi) *Suds-i Fakhri*, a sextant associated with the name of Fakhr al-Daulah Dailami.

Now we shall describe briefly the investigations carried out by the Muslim astronomers. Although the work of regular observations and construction of astronomical instruments was started as early as the second/eighth century by Ibrāhīm al-Fazāri (d. c. 180/796), the most brilliant period of Muslim astronomy commenced in the early part of the third/ninth century in the observatories constructed by the Caliph al-Māmūn (198–218/813–833). The observatory of Baghdād under Yaḥya bin abi Manṣūr (d. c. 216/831) made systematic observations of the heavenly bodies and found remarkably precise results for all the fundamental elements mentioned in Ptolemy's *Almajest*, such as the obliquity of the ecliptic, the precession of the equinoxes, the length

of the solar year. After recording these observations, Yaḥya compiled the celebrated "Tested Tables."[24] He was also the author of several works on astronomy.

Under the orders of al-Māmūn, the Muslim astronomers carried out one of the most delicate and difficult geodetic operations, the measuring of the arc of the meridian. The mean result gave 56 2/3 Arab miles as the length of a degree of meridian, which is a remarkably accurate value, for the Arabic mile is 6,473 ft. This value is equal to 366,842 ft., exceeding the real length of the degree between 38° and 36° latitudes by 2,877 ft.

Ḥabash al-Ḥāsib was an astronomer under al-Māmūn and al-Mu'taṣim; he compiled three astronomical tables, including the famous "Verified Tables." Apropos of the solar eclipse of 214/829, Ḥabash gave the first instance of a determination of time by an altitude which was generally adopted by the astronomers.[25]

'Ali bin 'Īsa al-Asṭurlābi was a famous maker of astronomical instruments. He took part in the degree measurement ordered by al-Māmūn, and wrote one of the earliest Arabic treatises on the astrolabe.[26]

Al-Marwarrudhi was one of those who took part in the solar observations made at Damascus in 217–218/832–833.[27]

The three sons of Mūsa bin Shākir made regular observations in the observatories in Baghdād between 236/850 and 257/870.[28]

Al-Farghāni was one of the most distinguished astronomers in the service of al-Māmūn and his successors. His famous work, *Kitāb fi Ḥarakāt al-Samāwiyyah wa Jawāmi' 'Ilm al-Nujūm* (Book on Celestial Motions and the Complete Science of the Stars), was translated into Latin in the sixth/twelfth century. It exerted marked influence on European astronomy. He accepted Ptolemy's theory and value of the precession but was of the view that it affected not only the stars but also the planets. He determined the diameter of the earth to be 6,500 miles, and found the greatest distances and also the diameters of the planets.[29]

Al-Mahāni (d. between 261/874 and 271/884) made a series of observations of lunar and solar eclipses and planetary conjunctions during the years 239–252/853–866; these were later used by ibn Yūnus.[30]

Al-Nairīzi (d. c. 310/922) compiled astronomical tables, made systematic observations, and wrote a book on atmospheric phenomena. He wrote a treatise on the spherical astrolabe which is very elaborate and is supposed to be the best Arabic work on the subject.[31]

[24] *Ibid.*
[25] Sarton, *op. cit.*, p. 565.
[26] *Ibid.*, p. 566.
[27] *Ibid.*
[28] *Encyclopaedia of Islam*, Vol. I, pp. 498 *et sq.*
[29] Sarton, *op. cit.*, p. 567.
[30] *Ibid.*, pp. 597–98.
[31] *Ibid.*, p. 599.

Thābit ibn Qurrah published solar observations, explaining his methods. He revised the theory of the movement of the sun.[32] To the eight Ptolemaic spheres, he made the addition of a ninth one *(primum mobile)* to account for the imaginary trepidation of the equinoxes, which was, however, later found to be an erroneous theory.[33]

Al-Battāni was one of the greatest astronomers of the Middle Ages. He wrote many books but his main work, the famous *De Numeris stellarum et motibus*, exerted great influence in Europe up to the time of the Renaissance. From 264/877 onwards he made astronomical observations of remarkable range and accuracy. His tables contain a catalogue of fixed stars for the year 267–68/880–81. He investigated the motion of the sun's apogee and found that its longitude had increased by 16° 47′ since the time of Ptolemy. This implied the discovery of the motion of the solar apsides, and of the slow variation in the equation of time. He determined many astronomical co-efficients with remarkable accuracy, and corrected the previous values of the precession of equinoxes and the obliquity of the ecliptic. He proved the possibility of the annular eclipses of the sun. He did not believe in the trepidation of the equinoxes, although the followers of Copernicus at a much later date did believe in it. Modern astronomy has shown that the Copernicans were wrong.[34] He determined the moon's nodes and discovered the wobbling motion of the earth's orbit.[35]

Ibn Amājur (abu Qāsim 'Abd Allah) together with his son abu al-Ḥasan 'Ali made many observations between 272/885 and 321/933 which were recorded by ibn Yūnus. They produced many astronomical tables, including the table of Mars according to Persian chronology.[36] Abu al-Ḥasan discovered that the moon's distance from the sun is not constant as assumed by Ptolemy.[37]

Al-Kūhi was the leading astronomer working in 378/988 at the Sharaf al-Daulah observatory.[38]

'Abd al-Raḥmān al-Ṣūfi (291–376/903–986) was one of the most eminent Muslim astronomers. His chief work, *Kitāb al-Kawākib al-Thābitah al-Muṣawwar* (Book of the Fixed Stars Illustrated), is regarded as one of the three masterpieces of Muslim observational astronomy, the other two being one by ibn Yūnus and a work prepared for Ulugh Beg.[39]

Ibn al-A'lam (d. 375/985) has been praised for the accuracy of his observations; his tables continued to be very popular for at least two centuries.[40] He determined the stellar motion by observing that the stars traverse one

[32] *Encyclopaedia of Islam*, Vol. I, p. 498.
[33] Sarton, *op. cit.*, p. 599.
[34] *Ibid.*, p. 603.
[35] *Iktifā' al-Qunū'*, p. 243.
[36] Sarton, *op. cit.*, p. 630.
[37] *Tamaddun-i 'Arab*, p. 420.
[38] Sarton, *op. cit.*, p. 665.
[39] *Ibid.*, p. 666.
[40] *Ibid.*

degree in seventy solar years.[41] He also determined the latitude and longitude of many stars,[42] and measured the greatest declination of the planet Mercury.[43] He found that the earth is spherical and may, therefore, be supposed to be inhabited everywhere.[44] He discovered the satellites of Jupiter, discussed the motion of the sun-spots, and determined the eccentric orbit of the comets.[45]

Abu al-Wafā' al-Būzjāni determined accurately the obliquity of the ecliptic in 344/955, and calculated the variation in the moon's motion. There is a difference of opinion about his discovery of the third liberation in the moon's motion. Some of the older writers believed that he discovered the third liberation and that Tycho Brahi rediscovered it in the tenth/sixteenth century.[46] But Sarton remarks that abu al-Wafā' did not discover this variation, but simply spoke of the second part of the evection, which is essentially different from the variation discovered by Tycho Brahi.[47]

Al-Khujandi made astronomical observations, including a determination of the obliquity of ecliptic, in Rayy, in 384/994.[48]

Maslamah ibn Aḥmad al-Majrīṭi (d. c. 398/1007) edited and corrected the astronomical tables of al-Khwārizmi replacing the Persian by the Arabic chronology. He wrote a treatise on the astrolabe and a commentary on Ptolemy's *Planisphaerium* both of which were later translated into Latin.[49]

Ibn Yūnus has been described by Sarton as the greatest Muslim astronomer. A well-equipped observatory in Cairo enabled him to prepare improved astronomical tables, called *al-Zīj al-Kabīr al-Ḥākimi*, completed in 398/1007. They describe observations of eclipses and conjunctions, old and new, and improved value of astronomical constants (obliquity of the ecliptic 23° 35′; longitude of the sun's apogee 86° 10′; solar parallax reduced from 3′ to 2′; precession of the quinoxes 51.2″ *per annum*), and give an account of the geodetic measurements made under al-Māmūn's orders.[50] He is specially noted for his method of longitude determination. As time difference is equivalent to longitude difference, the determination of local time at the same instant at two stations widely separated in longitude is sufficient. But there were no telegraphs or radio signals to give simultaneity. Ibn Yūnus proposed and used a signal from the moon—the first contact of a lunar eclipse. In this way he corrected many errors in longitudes in Ptolemy's geography.[51]

[41] *Sharḥ Chaghmani*, p. 23.
[42] *Iktifā' al-Qunū'*, p. 248.
[43] Ghulām Ḥusain Jaunpuri, *Jāmi' Bahādur Khāni*, p. 596.
[44] *Ibid.*, p. 668.
[45] *Ibid.*, p. 628.
[46] *Khulāṣah Tārīkh al-'Arab*, 243.
[47] Sarton, *op. cit.*, p. 666.
[48] *Ibid.*, p. 668.
[49] *Ibid.*
[50] *Ibid.*, p. 716.
[51] Carl Rufus, "The Influence of Islamic Astronomy in Europe and Far East," *Popular Astronomy*, May 1939, pp. 233–38.

Al-Bīrūni is regarded by Western historians of science as "one of the greatest scientists of all times whose critical spirit, toleration, love of truth, and intellectual courage were almost without parallel in medieval times."[52] He made accurate determination of latitudes and longitudes and also other geodetic measurements. He discussed in his book *Qānūn al-Mas'ūdi* for the first time the question that the earth rotates around its axis. The translation of the relevant Arabic passages is as follows: "When a thing falls from a height, it does not coincide with the perpendicular line of its descent, but inclines a little, and falls making different angles. When a piece of earth separates from it and falls, it has two kinds of motions: one is the circular motion which it receives from the rotation of the earth, and the other is straight which it acquires in falling directly to the centre of the earth. If it had only the straight motion, it would have fallen to the west of its perpendicular position. But since both of them exist at one and the same time, it falls neither to the west nor in the perpendicular direction, but a little to the east." This book of al-Bīrūni, viz., *al-Qānūn al-Mas'ūdi,* was written in 422/1030, and gave the true explanation of the rising and setting of the heavenly bodies as being due to the rotation of the earth, thus pointing to the error in the geocentric conception of the solar system. The heliocentric doctrine was not entirely unknown to the Arabs, who knew that the earth revolved round the sun and that the orbits of the planets were elliptic.[53] It should be noted that Copernicus gave the scientific formulation and detailed working out of the heliocentric theory some three centuries later.

Al-Zarqāli was "the best observer of his time. He invented an improved astrolabe called *Ṣafīḥah*; his description of it was translated into Latin, Hebrew, and many vernaculars. He was the first to prove explicitly the motion of the solar apogee with reference to the stars; according to his measurements it amounted to 12.04″ per year (the real value being 11.8″)." He edited the planetary tables called the "Toledan Tables."[54]

'Umar Khayyām was called to the new observatory of Rayy in 467/1074 by Sulṭān Malik Shāh Jalāl al-Dīn Saljūqi to reform the old calendar. Moritz Cantor remarks that the calendar prepared by 'Umar Khayyām, called *al-Tārīkh al-Jalāli*, was more accurate than any other proposed before or after his time. Its date was 10th Ramaḍān 471, i.e., 16th March 1079. The modern interpretation of Khayyām's calendar is that eight intercalary days should be introduced in thirty-three years, resulting in an error of one day in about 5,000 years. The Gregorian calendar leads to an error of one day in 3,330 years.[55]

Chingīz Khān erected a magnificent observatory at Marāghah near Tabrīz far surpassing any built by his predecessors. Naṣīr al-Dīn Ṭūsi was the greatest genius of this institution. He was quite original and independent, and criticized

[52] Sarton, *op. cit.*, p. 707.
[53] *Tamaddun-i 'Arab*, p. 425.
[54] Sarton, *op. cit.*, p. 758.
[55] *Ibid.*, p. 760.

Ptolemy quite severely, "paving the way for the overthrow of the geocentric system."[56]

Ulugh Beg, grandson of Tīmūr, established an observatory at Samarqand, Turkestan, in 823/1420, which was best equipped. A great work produced at this observatory was an independent star catalogue, known as the "Ulugh Beg Tables," based entirely upon new observations, the first in about sixteen hundred years, i.e., since the time of Hipparchus, second century B.C. The positions were given to the nearest minute of arc, and attained a high degree of precision for that period. Instruments used in this observatory are considered the best made up to that time.[57] It is said that his quadrant was so large that its diameter was equal to the height of the St. Sophia Church in Constantinople. This work on astronomy is regarded as one of the best books of the Muslim astronomers. It was written in 841/1437, and from it one can have a fair account of the knowledge possessed by the Muslims in the ninth/fifteenth century. The first part deals with the general principles of astronomy. The latter part contains the practical methods of calculating the lunar and solar eclipses and the construction of the tables and their applications; a list of the stars; the motion of the sun, the moon, and the planets; and the terrestrial latitudes and longitudes of the big cities of the world.[58]

The Mughuls inherited their fondness for astronomy from Ulugh Beg. Farishtah remarks that Humāyūn was a keen astronomer and spent a good deal of time in its pursuit.[59] An observatory was founded in Delhi under the orders of Muḥammad Shāh in 1137/1724, which was in the charge of the well-known mathematician Mirza Khair Allah. By this time the West had made great progress in astronomy as in other branches of knowledge, and therefore a commission consisting of the ablest men of the time was sent to Europe to study the new methods followed there and new results obtained through the then latest researches. The commission brought back with it some telescopes and other instruments and a few books prepared in Europe. The King of Portugal also deputed a European astronomer to go to Delhi with the commission. But when his data were checked at the Delhi observatory, local people detected errors and made corrections in his tables and calculations of the lunar and solar eclipses. This is ascribed to the fact that the instruments made in Europe at the time were of a smaller size than those available in the Delhi observatory.[60]

The Niẓāmīyyah observatory was erected at Hyderabad Deccan in the thirteenth/nineteenth century, and was the biggest institution of its kind in the East. It contained a sixteen-inch refracting telescope, a transit instrument, a Meridian circle, and a good deal of other equipment essential for a modern

[56] Carl Rufus, *op. cit.*, pp. 233–38.
[57] *Ibid.*
[58] *Tamaddun-i 'Arab*, pp. 422–24.
[59] *Tārīkh Farishtah*, p. 213.
[60] Ghulām Ḥusain Jaunpuri, *Jāmi' Bahādur Khāni.*

observatory. Its unique position was recognized by international organizations, and it had an important share in the preparation of the International Catalogue of Stars. After the establishment of the Osmania University, it became a constituent unit of that University.

The influence of the Muslims in this field is traceable from the many Arabic names and words that have become an integral part of the astronomical sciences. A long list of such words can be compiled, but it would be sufficient to mention a few: almanac *(al-munākh)*, almacantar *(al-muqanṭarah)*, nadir *(nādir)*, zenith *(samt al-rās)*, algol *(al-ghūl)*, altair *(al-ṭā'ir)*, aldebaran *(al-dabarān)*, fomalhaut *(fam al-ḥūt)*, denab *(dhanab)*, vega *(wāqi')*, and the various names of Muslim astronomers given to the craters of the moon.[61]

[61] Mohammad Abdul Rahman Khan, "Names of Thirteen Muslim Astronomers Given to Some Natural Features of the Moon," *Islamic Culture*, Vol. XXVII, No. 2, April 1953, pp. 78–85.

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Chapter LXIV

PHYSICS AND MINERALOGY

The Muslims contributed enormously to exact sciences such as mathematics, astronomy, physics, chemistry, botany, and zoology since they had succeeded in acquiring the knowledge of the sciences which had developed before the advent of Islam.

Abu Yūsuf Ya'qūb ibn Isḥāq al-Kindi[1] was the first Muslim scientist-philosopher. His pure Arabian descent earned him the title "The Philosopher of the Arabs." Indeed, he was the first and last example of an Aristotelian student in the Eastern Caliphate who sprang from the Arabian stock. His principal work on geometrical and physiological optics based on the optics of Euclid in Theon's recension was widely used both in the East and the West until it was

[1] See Chapter XXI.